PROFILE FOR STATISTICAL UNDERSTANDING ®

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The development of a Profile of Statistical Understanding is aimed at providing a tool to assist educators to identify what 'can be' expected of students rather than what 'should be'. This profile needs to cover all basic areas of statistics in such a way that specific profiles will identify what to expect of graduating secondary students with respect to 'statistical understanding' for tertiary education or the work force. Responses from 13 to 18 year-olds to open-ended questions were analyzed using the SOLO model as a framework for an hierarchy. The profile is presented and responses from typical students are discussed to elaborate on the categories. Rasch analysis combined the rankings of students on different questions to produce a measure of statistical understanding for each student. A profile for an average student is discussed.

INTRODUCTION

The development of a Profile of Statistical Understanding is aimed at providing a means of identifying stages within the development of statistical understanding. Many studies have identified 'lack of understanding' as a major cause of students' inability to think and reason statistically but when investigating statistical understanding Shaughnessy (1997) clearly identified the need to investigate what students 'can do'. This profile is intended to assist educators to identify what 'can be' expected of students rather than what 'should be'. The profile is still under development and further implementation and research is expected to assist in its refinement. Following a discussion of the study on which the profile is based and the structure of the profile, a measure of overall statistical understanding is introduced. Finally, an average student profile is presented based on this measure.

BASIS FOR PROFILE STRUCTURE

Holmes (1980) identified five basic areas of statistics: data collection; data tabulation and representation; data reduction; probability; and interpretation and inference. These five areas form one dimension of the profile, the other involves the various levels of statistical understanding. The research underpinning the framework of the profile involved a search for what students 'can do' in four of these areas. As yet, probability has not been included in the profile because it was not investigated in the study due to the large amount of research already being conducted in that area. Probability does, however, need to be included in the profile in the future.

Investigations involved 180 students, aged from 13 to 18, from Years 7 to 12 in an Australian secondary school, who responded to open-ended questions in each of the four areas. Using the SOLO model as a framework, student responses were ranked into hierarchies of statistical understanding within each of the four areas. Students were asked to give reasons for their responses which were included in the analysis. SOLO was found to be a useful guide in developing the hierarchy which provided the basis for developing the profile.

More detailed discussions of the results of the study (Reading, 1996) are available: data collection (Reading, 2001); data tabulation and representation (Reading, 1999); data reduction (Reading & Pegg, 1996); and interpretation and inference (Reading, 1998). Cycles of levels of responses from both the iconic and the concrete-symbolic modes of the SOLO model were identified, within each of the four areas. Unique aspects of the responses for each level within these cycles form the descriptors for the profile.

PROFILE OF STATISTICAL UNDERSTANDING

The profile is designed as a tool to assist educators to assess a student's level of statistical understanding. Use of the profile is not task specific, that is, a response to any task or problem involving statistics could be assessed using the profile. This eliminates the need for having to administer specific tasks to assess student understanding.

SOLO Mode	*General*	*Data Collection*	*Data Tabulation and Representation*	*Data Reduction*	*Interpretation and Inference*
Ikonic	do not see the value of data in an argument – don't interact with the data	suggest using an already given result rather than collect data	only mention the title or the axis labels (variables) involved	reduce data but to a value not directly linked to the data	describe patterns but prediction impossible
Concrete-symbolic cycle 1	focus on individual data items - variation not seen as global	concern with physical aspects of data collection such as time and cost	interact with the data in non-statistical terms – describe individual data points	reduce data to a more useable form based on individual data items	describe patterns but predict using personal experience and not the data
Concrete-symbolic cycle 2	focus on features of the data - variation seen as global	concern for quality or accuracy of he data collected- identify relevant variables for consideration	interact with the data in more statistical terms - describe features or behaviour of the data	reduce data to a more useable form based on features of the data	predict using pattern descriptions - justification makes use of the data

Figure 1. Profile of Statistical Understanding.

The first column in the profile (see Figure 1) identifies the relevant SOLO mode or cycle within a mode. Each student response was assigned a level on the hierarchy from 0 to 8. The ionic mode contains levels 0, 1 and 2 and cycle 1 in the concrete-symbolic mode contains levels 3, 4 and 5, while cycle 2 contains levels 6, 7 and 8. Each cycle of three levels represents the unistructural, multistructural, and relational levels of the SOLO Model. The General attribute, in the second column, gives an overview of the approach to data, which is a common theme across each mode/cycle. The last four columns describe the student approach in each of the four areas. More detailed descriptions of the features of responses in these various categories can be found in the papers identified earlier.

The profile is in an early stage of its development, needing both expansion and refinement. Currently, the categories identified have been based on Secondary student responses. Vertical expansion would result from investigating responses from Primary, aged 5 to 12, students and Tertiary, aged over 18, students. Horizontally, the profile needs to be expanded by adding the fifth area, probability.

TYPICAL STUDENT PROFILES

Two typical student profiles, one for Year 7 and one for Year 12, are now presented in terms of actual responses (Figure 2), to elaborate on the profile. Year 7, age 13, was chosen as these are the students just entering their secondary education and Year 12, age 18, was chosen as they are completing their secondary experience of 'statistics' and heading off for tertiary study or the workforce. A 'typical' student response guide in the profile is created from the students' responses, in that year, at the most common level. Many students in Year 7 are responding in Cycle 1 of the Concrete-symbolic mode, so the typical student profile for Year 7 has been built from responses given by Year 7 students in Cycle 1. Most students in Year 12 are responding in Cycle 2 of the Concrete-symbolic mode and the typical student profile for Year 12 has been built from responses given by Year 12 students in Cycle 2. In Figure 2 the actual words from student responses are presented in italics, with some explanatory words non-italicised, but for the sake of brevity the responses have sometimes been truncated from the original. It should be noted that these students were given very open-ended questions and if a task is designed so that students are more specifically directed to consider one particular aspect of a situation, then the response may be at a level higher than expected due to the prompting.

Typical Student	*Data Collection*	*Data Tabulation and Representation*	*Data Reduction*	*Interpretation and Inference*
Year 7 *(Cycle 1)*	do a survey and get people to ring the radio station…becaus e it would be easy for everyone	how many icecreams she ate each week for seven weeks	26-33 so you don't have to write so much	*low to high to low* as pattern description, predicting *1999* as the next year when number of accidents exceeds 8 *because its good*
Year 12 *(Cycle 2)*	collect people of varying ages and backgrounds …. not biased to any group	on average the student eats 4.7 icecreams per week, with a total of 33 icecreams	26-33 that is the range	*every 4th year the numbers of accidents decreases dramatically* as pattern description, predicting *1992,1993 because the accidents follow a pattern and usually every 3rd or 4th year they exceed 8 accidents, so I just followed the pattern*

Figure 2. Typical Student Responses.

Now consider the profile of the typical Year 7 student in more detail. When presented with a task to suggest a data collection procedure and justify it, although able to suggest taking a survey, the only justification was ease of collection and no concern was shown for the accuracy or quality of the data collected. When asked to describe tabulated data this student did not discuss anything beyond the information contained in the heading or the axes' labels. When asked to reduce data a simple summary of the data was produced, the smallest and largest, but no actual 'statistic', nor explanation with reference to the data, was given. In this case, a reason for reducing was given, rather than a reason why 26-33 was chosen. When asked to describe a pattern in the data and make a prediction, this student gave an overall impression of the data, trying to explain the changing numbers as low, high and then low again. Although a reasonable prediction was made the student only said that it was 'good' and did not attempt to justify the prediction in terms of the data, or the pattern observed. Overall, this student presents as being aware of, but not able to engage with, the data; not yet sufficiently aware of the importance of the variation in data; nor capable of discussing the behaviour of features of data in statistical terms.

Considering the typical Year 12 student, the sample suggested in the data collection question took into consideration two variables 'background' and 'age' and the student identified that such a process was necessary to ensure that the data was not 'biased', so recognizing the need for concern with the quality of the data. When describing tabulated data the student suggested the average and total as a means of description, indicating the ability not just to read, but also interpret, the data presented. When reducing data the student gave the range as a response. Finally, when describing the pattern this student was able to pick up on, and describe in a concise manner, a cyclic pattern that was evident in the data. However, more importantly, the student made a prediction and justified the prediction using the pattern identified in the data.

These two profiles indicate a shift in the level of understanding of students from Year 7 through to Year 12. This shift from Cycle 1 to Cycle 2 is a transition, which needs to be nurtured by those presenting students with activities involving data. However, it is not being suggested here that the examples presented should be typical of students aged 13 years and 18 years in other countries. It should be noted that in most Australian schools students do not study specific statistics courses and only meet statistical concepts in their mathematics courses. In fact, these data were collected in New South Wales, the State with the least statistics in its mathematics courses in secondary school. What is important, though, is that the profile allows a method of describing the stage students have reached in their statistical understanding, irrespective of the amount of statistics experienced in structured courses.

RASCH ANALYSIS

The Rasch model was used to produce an estimate of statistical understanding on a logit scale, for each student, which incorporates the information from the questions covering all four focus areas (Reading, 1996, pp. 93-94). Masters (1982, pp.157-158) partial credit Rasch model for polychotomous data, where responses can be coded according to an increasing, or decreasing, degree of 'correctness', considers the individual difficulty of each successive step from one level to the next in the question, using a formula for calculating the probability of a student responding at a particular level of a particular question. The advantage of the partial credit model is that the parameters are separable, making it possible to produce sufficient statistics for person ability (understanding) and for step difficulty within each question. Masters (1982, pp.163-166) used a maximum likelihood procedure to estimate the parameters, overall ability for each student and difficulty for each question, in the model. Appropriate measures indicated that the questions were all consistent on testing the same underlying construct, statistical understanding, and that the data fitted the Rasch model well (Reading, 1996, p170). Although the study reported on measures of overall understanding and question difficulty (Reading, 1996, pp. 170-190), this paper only discusses the measure of student understanding.

MEASURE OF OVERALL STATISTICAL UNDERSTANDING

The spread of estimates of overall statistical understanding is shown in Figure 3. Each X represents one student and the - - - line represents the mean estimate, 0.2, for all students. The spread of estimates is reasonably well balanced, except for the Year 12 student with 21.6.

```
 2.2    X
 2.1
 2.0
 1.9
 1.8    XX
 1.7
 1.6    XXX
 1.5
 1.4    X
 1.3
 1.2    X
 1.1
 1.0    XXXX
 0.9    XXXXXXX
 0.8    XXXXXXXX
 0.7
 0.6    XXXXXXXXXXXXXXXXX
 0.5    XXXXX
 0.4    XXXXXXXXX
 0.3    XXXXXXXXXXXXXXXXXX
 0.2    XXXXXXXXXXXX - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - Overall Mean
 0.1    XXXXXXXXXXXXXXX
 0.0    XXXXXXXXXXXXXXXXXXXXXXXXXX
-0.1    XXXXXXX
-0.2    XXXXXXXXXXXXXX
-0.3    XXXXXXXXXXXXXX
-0.4    XXXXX
-0.5    XXXX
-0.6    XXX
-0.7
-0.8    X
-0.9
-1.0    XXX
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Figure 3. Estimates of Understanding.

An analysis of variance was performed on these estimates to test the overall influence of various factors including academic year (Reading, 1999, pp. 172-173), and results indicated an increased understanding with successive academic years, as would be expected. However, pair-wise comparisons (Reading, 1996, pp. 298-299) showed that the only significant differences were Year 11 to Year 7, and Year 12 to each of the other years. Hence, although there was a significant difference between the academic years in terms of understanding, the difference between successive years is mostly not significant. This indicates that for these students most years of secondary schooling elapsed before there was a significant change in statistical understanding, the most significant change being between Years 11 and 12. Although this measure of overall statistical understanding is useful for comparison of students or cohorts, more useful information for assessing students' statistical understanding and mapping their progression is gained by using the profile.

PROFILE OF AN AVERAGE STUDENT

Where does this measure of understanding place the *average* secondary student, as far as statistical understanding is concerned? Figure 4 was constructed for four questions, one from each area, using level thresholds (Reading, 1996, pp. 178-181), which are estimates of the score that a student would need to have a 50% chance of being coded at a particular level on a particular question. For example, the Level 5 at 0.2 for Representation, indicates that the threshold value for Level 5 is 0.2, that is, a student with a measure of understanding of 0.2 has a 50% chance of responding at Level 5 on a Representation task. The position of the 'average' student is shown using a - - - line, the overall average at 0.2, the Year 7 average at –0.1 and Year 12 average at 0.7.

	Collection	Representation	Reduction	Interpretation
1.3	6		7	
1.2				
1.1				7
1.0		7		
0.9	5			
0.8				
0.7 Year 12	- - - - - -	- - - - - -	- - - - - -	- - - - - -
0.6		6		
0.5				
0.4				
0.3				
0.2 Average	- - - - - -	- - 5 - -	- - - - - -	- - - - - -
0.1		4		
0.0				
-0.1 Year 7	4 - - - -	- - - - - -	- - - - - -	- - 6 - -
-0.2		3	6	5
-0.3				4
-0.4			5	3
-0.5			4	
-0.6			3	2
-0.7				
-0.8	3			
-0.9			2	
-1.0				
-1.1				
-1.2				
-1.3	2			
-1.4				
-1.5				
-1.6				
-1.7		2		

Figure 4. Threshold Values for Levels in the Profile of Understanding.

Of most interest are the thresholds for Level 3 and Level 6, which are the entry points to the first and the second cycles, respectively, of the concrete-symbolic mode. The diagram shows the entry into Level 3 is similar for the four areas. However, the entry estimates for Level 6 show much more variation. It appears that a much lower overall estimate of understanding is exhibited by a student entering into the second cycle in data reduction and interpretation, than for data collection and representation. A measure of more than –0.8 (see Figure 3) for most students indicates that during Secondary schooling the minimum level of response expected should be moving into Cycle 1 of the Concrete-symbolic mode.

Considering how this profile varies over academic year, the average Year 7 student is operating in the first cycle of the concrete-symbolic mode in most areas and just on the point of entering the second cycle in data reduction and interpretation. Through Years 8 to 11 the average student progresses much more rapidly in representation, than in the other areas. In fact, it is not until Year 12 that the average student begins to enter the second cycle in the focus area of data collection. The overall effect is that, over the six years of schooling, many students move from the first cycle to the second cycle in the concrete-symbolic mode, engaging with data and beginning to recognize the need to use data to justify their responses.

CONCLUSION

Although the profile is still in its infancy, aspects are useful in assessing students' understanding. Recently, the profile proved useful in a research project to identify the level of understanding before and after a statistically related task. More generally, teachers could use the profile as a means of recording a students' progress in statistical understanding during their schooling, to be passed on from one year to the next. An anticipated important role for the profile is as a basis for describing what to expect from the average student in a specific cohort of students with respect to 'statistical understanding'. Such information could be useful for tertiary educators and employers.

However, to produce a more effective profile more research is needed. There is still a need to explore responses at the Primary and Tertiary level and to develop levels of understanding for probability. Also, an understanding of variation is fundamental to all aspects of statistics, so there is a need to explore the understanding of variation and its role in the profile.

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